Eur. Phys. J. C 14, 479–485 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000368

THE EUROPEAN PHYSICAL JOURNAL C
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Springer-Verlag 2000

Test of CPT symmetry in cascade decays*

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Received: 14 July 1999 / Revised version: 12 February 2000 /
Published online: 14 April 2000 – © Springer-Verlag 2000

Abstract. Cascade mixing provides an place for elegant study of B^0–$\bar{B}^0$ mixing. We use this idea to study the CPT violation caused by B^0–$\bar{B}^0$ mixing. An approximation method is adopted to treat the two complex B^0–$\bar{B}^0$ mixing parameters θ and ϕ. A procedure to extract the parameters θ and ϕ is suggested. The feasibility of exploring CPT violation and determining θ and ϕ in the future B-factories and LHC-B is discussed.

1 Introduction

The violation of the three discrete symmetries C, P and T have changed our ideas about the physical world. The discovery of CP violation in the K^0–$\bar{K}^0$ complex was a long time ago. Recently, direct T violation [1] and direct CP violation ($\mathrm{Re}(\epsilon'/\epsilon) \neq 0$) [2] have been found experimentally. Only the combined CPT symmetry is left unbroken. The CPT theorem is a general result of the local, relativistic field theory. If it is violated, it will cause a fundamental crisis in our present particle theory. The recent tests of CPT violating effects give the bounds [3] $r_K \equiv | (m_{\bar{K}^0} - m_{K^0})/(m_{K^0}) | \leq 10^{-18}$ and the very uncertain values $\mathrm{Re}(\delta) = (3.0 \pm 3.3_{\rm stat} \pm 0.6_{\rm syst}) \times 10^{-4}$ and $\mathrm{Im}(\delta) = (-1.5 \pm 2.3_{\rm stat} \pm 0.3_{\rm syst}) \times 10^{-2}$. Tests of CPT symmetry in the B system have been suggested on the basis of theory [4] and phenomenology [5,6]. In [5], the authors point out that CPT violation in B^0–$\bar{B}^0$ mixing can lead to a dilepton asymmetry of neutral-B decays. They also discuss some general effects caused by CPT violation.

Neutral-meson interferometry is a powerful tool for investigating discrete symmetry. CP violation (ϵ), direct T and direct CP violation are all observed in the K^0–$\bar{K}^0$ complex. In the decay chain $B \to J/\psi + K \to J/\psi + [f]$, neutral K mixing follows on the heels of B mixing. This mixing is called "cascade mixing" and the decay is "cascade decay". Cascade mixing has attracted some interest of theorists [7,8]. The extension to the decay chain $B \to D \to [f]_D$ for exploring new physics can be found in [9]. The advantage of cascade decay is that we can use the known K mixing parameter to determine the B mixing parameter. In [8], Kayser shows that the cascade decay contains more information than the usually discussed processes. So cascade decay provides a complex and elegant window for the details of the B^0–$\bar{B}^0$ mixing.

* Supported in part by National Natural Science Foundation of China

In this work, we shall make a detailed study of the CPT violating effects caused by B^0–$\bar{B}^0$ mixing in cascade decays. B^0–$\bar{B}^0$ mixing is described by two complex phases, θ and ϕ. We first choose an approximation method to treat the parameters θ and ϕ and give the general formulas of the direct CPT and T asymmetry. Then we investigate how to extract the B^0–$\bar{B}^0$ mixing parameters. The feasibility of exploring CPT violation in B-factories and LHC-B is discussed. Finally, we suggest a procedure to determine the two complex phases.

2 B^0–$\bar{B}^0$ mixing and CP, T, CPT asymmetries

The weak interaction can cause oscillations between B^0 and $\bar{B}^0$. The eigenstates of the weak decays are not B^0 and $\bar{B}^0$ but superpositions of them which have simple exponential laws. The two eigenstates of the B^0–$\bar{B}^0$ mesons are given by

$$\begin{aligned} |B_L\rangle &= \frac{1}{\sqrt{|p_1^2| + |q_1|^2}} [p_1|B^0\rangle + q_1|\bar{B}^0\rangle], \\ |B_H\rangle &= \frac{1}{\sqrt{|p_2^2| + |q_2|^2}} [p_2|B^0\rangle - q_2|\bar{B}^0\rangle], \end{aligned} \quad (1)$$

and their eigenvalues are

$$\begin{aligned} \mu_L &= m_L - \frac{\mathrm{i}}{2}\Gamma_L = m_B - \frac{\Delta m_B}{2} \\ &\quad - \frac{\mathrm{i}}{2}\left(\Gamma_B + \frac{\Delta\Gamma_B}{2}\right) \\ &= m_B - \frac{\mathrm{i}}{2}\Gamma_B - \frac{\Delta m_B}{2} - \frac{\mathrm{i}}{2} y\Gamma_B, \\ \mu_H &= m_H - \frac{\mathrm{i}}{2}\Gamma_H = m_B + \frac{\Delta m_B}{2} \end{aligned}$$

$$- \frac{\mathrm{i}}{2}\left(\Gamma_B - \frac{\Delta\Gamma_B}{2}\right)$$
$$= m_B - \frac{\mathrm{i}}{2}\Gamma_B + \frac{\Delta m_B}{2} + \frac{\mathrm{i}}{2}y\Gamma_B. \quad (2)$$

From PDG98 [10], $x \equiv \Delta m_B/\Gamma_B \sim 0.7$, while $y \equiv \Delta\Gamma_B/(2\Gamma_B) \leq 10^{-2}$ is theoretically expected [11].

The mixing parameters p_i, q_i are related by[12]

$$\frac{q_1}{p_1} = \mathrm{tg}\frac{\theta}{2}\mathrm{e}^{\mathrm{i}\phi}, \quad \frac{q_2}{p_2} = \mathrm{ctg}\frac{\theta}{2}\mathrm{e}^{\mathrm{i}\phi}, \quad (3)$$

where θ and ϕ are complex phases in general. For real θ and ϕ, $0 < \theta < \pi$, $0 \leq \phi < 2\pi$.

From (1), the mass difference between B^0 and $\bar{B}^0$ is

$$M_{B^0} - M_{\bar{B}^0} = (\mu_L - \mu_H)\frac{p_1q_2 - p_2q_1}{p_1q_2 + p_2q_1} = (\mu_L - \mu_H)\cos\theta, \quad (4)$$

where $M_{\overset{(-)}{B^0}} = m_{\overset{(-)}{B^0}} - (\mathrm{i}/2)\Gamma_{\overset{(-)}{B^0}}$u.

Using the Bell–Steinberg unitarity relation [13], we have

$$|\langle B_H|B_L\rangle| \leq \frac{\Gamma_L\Gamma_H}{|\mu_L^* - \mu_H|} = \frac{\sqrt{1-y^2}\Gamma_B}{|\mathrm{i}-x|\Gamma_B} \simeq 0.8. \quad (5)$$

This constraint is more relaxed than that in the K^0–$\bar{K}^0$ complex where $|\langle K_L|K_S\rangle| \leq 0.06$. So, $|B_H\rangle$ and $|B_L\rangle$ are likely unorthogonal. The more relaxed constraint of (5) perhaps indicates a large *CP* or *CPT* violation in the B^0 system.

The initial $|B^0\rangle$ or $|\bar{B}^0\rangle$ will evolve after a proper time t to

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \bar{g}_+(t)|\bar{B}^0\rangle,$$
$$|\bar{B}^0(t)\rangle = g_-(t)|\bar{B}^0\rangle + \bar{g}_-(t)|B^0\rangle, \quad (6)$$

where

$$g_+(t) = f_+(t) + \cos\theta f_-(t),$$
$$g_-(t) = f_+(t) - \cos\theta f_-(t),$$
$$\bar{g}_+(t) = \sin\theta\mathrm{e}^{\mathrm{i}\phi}f_-(t),$$
$$\bar{g}_-(t) = \sin\theta\mathrm{e}^{-\mathrm{i}\phi}f_-(t), \quad (7)$$

and

$$f_+(t) = \frac{1}{2}(\mathrm{e}^{-\mathrm{i}\mu_L t} + \mathrm{e}^{-\mathrm{i}\mu_H t})$$
$$= \mathrm{e}^{-\mathrm{i}m_B t - \frac{1}{2}\Gamma_B t}\mathrm{ch}\left(\frac{\mathrm{i}x - y}{2}\Gamma_B t\right),$$
$$f_-(t) = \frac{1}{2}(\mathrm{e}^{-\mathrm{i}\mu_L t} - \mathrm{e}^{-\mathrm{i}\mu_H t})$$
$$= \mathrm{e}^{-\mathrm{i}m_B t - \frac{1}{2}\Gamma_B t}\mathrm{sh}\left(\frac{\mathrm{i}x - y}{2}\Gamma_B t\right). \quad (8)$$

The probability for $|B^0\rangle$ to transform in a proper time t into $|B^0\rangle$ is

$$P_{B^0(t)\to B^0} = \left|\langle B^0|B^0(t)\rangle\right|^2 = |g_+(t)|^2. \quad (9)$$

Similarly, we can define $P_{B^0(t)\to\bar{B}^0}$, $P_{\bar{B}^0(t)\to B^0}$, and $P_{\bar{B}^0(t)\to\bar{B}^0}$.

So the mixing-induced *CPT* and *T* asymmetries can be defined as

$$A_{CPT}(t) \equiv \frac{P_{B^0(t)\to B^0} - P_{\bar{B}^0(t)\to\bar{B}^0}}{P_{B^0(t)\to B^0} + P_{\bar{B}^0(t)\to\bar{B}^0}}$$
$$= \frac{2\mathrm{Re}\left[\cos\theta\mathrm{sh}\left(\frac{\mathrm{i}x-y}{2}\Gamma_B t\right)\left(\mathrm{ch}\left(\frac{\mathrm{i}x-y}{2}\Gamma_B t\right)\right)^*\right]}{\left|\mathrm{ch}\left(\frac{\mathrm{i}x-y}{2}\Gamma_B t\right)\right|^2 + |\cos\theta|^2\left|\mathrm{sh}\left(\frac{\mathrm{i}x-y}{2}\Gamma_B t\right)\right|^2} \quad (10)$$

and

$$A_T(t) \equiv \frac{P_{B^0(t)\to\bar{B}^0} - P_{\bar{B}^0(t)\to B^0}}{P_{B^0(t)\to\bar{B}^0} + P_{\bar{B}^0(t)\to B^0}} = \frac{|\mathrm{e}^{\mathrm{i}\phi}|^2 - |\mathrm{e}^{-\mathrm{i}\phi}|^2}{|\mathrm{e}^{\mathrm{i}\phi}|^2 + |\mathrm{e}^{-\mathrm{i}\phi}|^2}. \quad (11)$$

Some analysis can lead to the following conclusions [14]:
(1) *CPT* invariance requires $\cos\theta = 0$, and thus $\theta = \pi/2$.
(2) T invariance requires $\phi = 0$;
(3) CP conservation requires $\cos\theta = 0$ (and thus $\theta = \pi/2$) and $\phi = 0$.

In the standard model, *CPT* is conserved and $|q/p| - 1 = |\mathrm{e}^{\mathrm{i}\phi}| - 1 = (1/2)\mathrm{Im}\Gamma_{12}/M_{12} \sim \mathcal{O}(10^{-3})$ [21]. Thus, the direct *T* violation mixing is about the order of $\mathcal{O}(10^{-3})$. From our experience in the *K* system, we guess that the *CPT* violating effects may be smaller than the *CP* violating effects.

Under the above assumption, it is convenient to introduce a complex θ' and two real ϕ_0, ϕ' by

$$\theta = \frac{\pi}{2} + \theta', \qquad \theta' = \mathrm{Re}\theta' + \mathrm{i}\mathrm{Im}\theta',$$
$$\phi = \phi_0 + \mathrm{i}\phi', \quad \phi_0 = \mathrm{Re}\phi,\ \phi' = \mathrm{Im}\phi, \quad (12)$$

where the $\mathrm{Re}\theta'$, $\mathrm{Im}\theta'$, ϕ_0 and the ϕ' are all real, and $|\theta'| << 1$, $|\phi'| << 1$. The relation between ϕ_0 and the CKM phase β is $\phi_0 = -2\beta$.

Then we have a very simple relation:

$$\cos\theta = -\theta', \quad \sin\theta = 1, \quad \mathrm{e}^{\mathrm{i}\phi} = \mathrm{e}^{\mathrm{i}\phi_0}(1 - \phi'). \quad (13)$$

Here we only keep terms up to the first order of θ' and ϕ'.

From PDG98 [10], the mixing parameter $\Delta\Gamma_B/\Gamma_B$ has not been measured experimentally up to now. We further assume $y = 0$ in order to simplify the formulation below. Thus,

$$\mathrm{ch}\left(\frac{\mathrm{i}x - y}{2}\Gamma_B t\right) = \cos\frac{\Delta m_B t}{2},$$
$$\mathrm{sh}\left(\frac{\mathrm{i}x - y}{2}\Gamma_B t\right) = \mathrm{i}\sin\frac{\Delta m_B t}{2}. \quad (14)$$

From (10), (11), (13) and (14), we obtain

$$A_{CPT}(t) = \frac{2\mathrm{Im}\theta'\sin\Delta m_B t}{1 + \cos\Delta m_B t},$$
$$A_T(t) = -2\phi'. \quad (15)$$

So the mixing-induced *CPT* asymmetry is proportional to $\mathrm{Im}\theta'$, and the mixing-induced T asymmetry is proportional to $\mathrm{Im}\phi = \phi'$.

Now, we discuss two cases:

(1) The final state is not a CP eigenstate.

We study the decays $B^0 \to Xl^+\nu$, $\bar{B}^0 \to \bar{X}l^-\nu$. From the $\Delta B = \Delta Q$ rule, the decays of $B^0 \to \bar{X}l^-\nu$, $\bar{B}^0 \to Xl^+\nu$ are forbidden.

For the allowed processes, we define the amplitude

$$\langle Xl^+\nu|H|B^0\rangle = A, \quad \langle \bar{X}l^-\nu|H|\bar{B}^0\rangle = A^*.$$

So the asymmetries of the semileptonic decay rates are

$$\begin{aligned} D_1(f,t) &\equiv \frac{\Gamma(B^0(t)\to Xl^+\nu)-\Gamma(\bar{B}^0(t)\to \bar{X}l^-\nu)}{\Gamma(B^0(t)\to Xl^+\nu)+\Gamma(\bar{B}^0(t)\to \bar{X}l^-\nu)} \\ &= \frac{P_{B^0(t)\to B^0}-P_{\bar{B}^0(t)\to\bar{B}^0}}{P_{B^0(t)\to B^0}+P_{\bar{B}^0(t)\to\bar{B}^0}} = A_{CPT}(t), \\ D_2(f,t) &\equiv \frac{\Gamma(B^0(t)\to \bar{X}l^-\nu)-\Gamma(\bar{B}^0(t)\to Xl^+\nu)}{\Gamma(B^0(t)\to \bar{X}l^-\nu)+\Gamma(\bar{B}^0(t)\to Xl^+\nu)} \\ &= \frac{P_{B^0(t)\to \bar{B}^0}-P_{\bar{B}^0(t)\to B^0}}{P_{B^0(t)\to \bar{B}^0}+P_{\bar{B}^0(t)\to B^0}} = A_T(t). \end{aligned} \quad (16)$$

Equation (16) shows that *CPT* and T asymmetry can lead to CP asymmetry. One can use the CP asymmetry of semileptonic B decays to measure the *CPT* and T violation parameter.

(2) Final state is CP eigenstate.

The decay rate for an initial B^0 or $\bar{B}^0$ transforming into a CP eigenstate f is

$$\begin{aligned} &\Gamma(B^0(t)\to f) = \mathrm{e}^{-\Gamma_B t}|A|^2\left\{\frac{1+\cos\Delta m_B t}{2}\right. \\ &+\mathrm{Im}\theta'\sin\Delta m_B t + \left|\frac{\bar{A}}{A}\right|^2(1-2\phi')\frac{1-\cos\Delta m_B t}{2} \\ &-\mathrm{Im}\left[\mathrm{e}^{\mathrm{i}\phi_0}\frac{\bar{A}}{A}(\sin\Delta m_B t-\phi'\sin\Delta m_B t\right. \\ &\left.\left.+\mathrm{i}\theta'^*(1-\cos\Delta m_B t))\right]\right\}, \quad (17) \\ &\Gamma(\bar{B}^0(t)\to f) = \mathrm{e}^{-\Gamma_B t}|A|^2\left\{(1+2\phi')\frac{1-\cos\Delta m_B t}{2}\right. \\ &+\left|\frac{\bar{A}}{A}\right|^2\left(\frac{1+\cos\Delta m_B t}{2}-\mathrm{Im}\theta'\sin\Delta m_B t\right) \\ &+\mathrm{Im}\left[\mathrm{e}^{\mathrm{i}\phi_0}\frac{\bar{A}}{A}(\sin\Delta m_B t+\phi'\sin\Delta m_B t\right. \\ &\left.\left.+\ \mathrm{i}\theta'(1-\cos\Delta m_B t))\right]\right\}, \end{aligned}$$

where $A \equiv A(B^0 \to f)$ and $\bar{A} \equiv A(\bar{B}^0 \to f)$.

For $f = J/\psi K_S$, $\bar{A}/A = -1$, the CP asymmetry is

$$\begin{aligned} &D(J/\psi K_S,t) = \\ &\frac{\Gamma(B^0(t)\to J/\psi K_S)-\Gamma(\bar{B}^0(t)\to J/\psi K_S)}{\Gamma(B^0(t)\to J/\psi K_S)+\Gamma(\bar{B}^0(t)\to J/\psi K_S)} \\ &= \sin\phi_0\sin\Delta m_B t + \mathrm{Re}\theta'\cos\phi_0(1-\cos\Delta m_B t) \\ &+\mathrm{Im}\theta'\sin\Delta m_B t - \mathrm{Im}\theta'\sin\phi_0(1-\cos\Delta m_B t) \\ &-\phi'(1-\cos\Delta m_B t)+\phi'\sin\phi_0\sin\Delta m_B t. \end{aligned} \quad (18)$$

For $f = J/\psi K_L$, $\bar{A}/A = +1$, the CP asymmetry is

$$\begin{aligned} &D(J/\psi K_L,t) \\ &= \frac{\Gamma(B^0(t)\to J/\psi K_L)-\Gamma(\bar{B}^0(t)\to J/\psi K_L)}{\Gamma(B^0(t)\to J/\psi K_L)+\Gamma(\bar{B}^0(t)\to J/\psi K_L)} \\ &= -\sin\phi_0\sin\Delta m_B t - \mathrm{Re}\theta'\cos\phi_0(1-\cos\Delta m_B t) \\ &+\mathrm{Im}\theta'\sin\Delta m_B t + \mathrm{Im}\theta'\sin\phi_0(1-\cos\Delta m_B t) \\ &-\phi'(1-\cos\Delta m_B t)-\phi'\sin\phi_0\sin\Delta m_B t. \end{aligned} \quad (19)$$

3 Cascade decays

We have discussed the B^0–$\bar{B}^0$ mixing in the previous section. Now we turn to the cascade mixing involving both neutral B and neutral K systems in succession. Neglecting *CPT* violating effects in the neutral K system, the weak eigenstates of the neutral K mesons can be represented in the usual form:

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{|p_K^2|+|q_K|^2}}[p_K|K^0\rangle + q_K|\bar{K^0}\rangle], \\ |K_L\rangle &= \frac{1}{\sqrt{|p_K^2|+|q_K|^2}}[p_K|K^0\rangle - q_K|\bar{K^0}\rangle], \end{aligned} \quad (20)$$

and their eigenvalues are

$$\mu_{S(L)} = m_K \overset{(+)}{-} \frac{\Delta m_K}{2} - \mathrm{i}\frac{\Gamma_{S(L)}}{2}, \quad (21)$$

where m_K is the average of the K_S and K_L masses, $\Gamma_{S,L}$ are the $K_{S,L}$ widths.

The time evolution of the neutral K mesons can easily be obtained:

$$\begin{aligned} |K^0(t)\rangle &= e_+(t)|K^0\rangle + \frac{q_K}{p_K}e_-(t)|\bar{K^0}\rangle, \\ |\bar{K^0}(t)\rangle &= \frac{p_K}{q_K}e_-(t)|K^0\rangle + e_+(t)|\bar{K^0}\rangle, \end{aligned} \quad (22)$$

where

$$e_\pm(t) = \frac{1}{2}(\mathrm{e}^{-\mathrm{i}\mu_S} \pm \mathrm{e}^{-\mathrm{i}\mu_L}). \quad (23)$$

Consider the decay chain $B \to J/\psi + K \to J/\psi + [f]$ where f can be 2π, 3π and $\pi l\nu$ as shown in Fig. 1. Other decay modes of the neutral K mesons are neglected because of either there being only very small fractions or of being of less physical interest for this paper. We first give the formulation of the most complicated case where the final state $f = \pi l\nu$.

According to [15], the decay amplitude of the cascade decay $B^0 \overset{t_1}{\to} J/\psi + K \overset{t_2}{\to} J/\psi + [\pi^- l^+\nu]$ is

$$\begin{aligned} &A(B^0 \overset{t_1}{\to} J/\psi + K \overset{t_2}{\to} J/\psi + [\pi^- l^+\nu]) \quad (24) \\ &= g_+(t_1)A(B^0\to J/\psi K^0)e_+(t_2)A(K^0\to\pi^-l^+\nu) \\ &+\bar{g}_+(t_1)A(\bar{B}^0\to J/\psi\bar{K^0})\frac{p_K}{q_K}e_-(t_2)A(K^0\to\pi^-l^+\nu). \end{aligned}$$

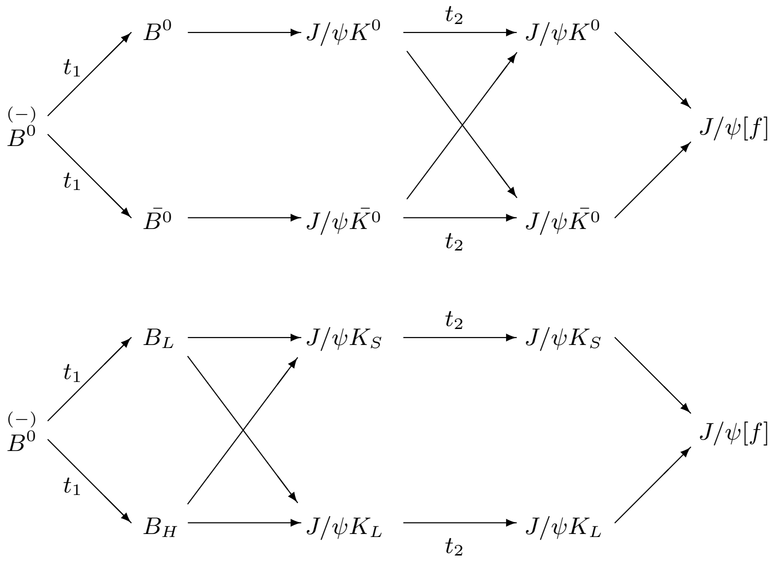

Fig. 1. The cascade decay chains $B \to J/\psi K \to J/\psi[f]$

We assume that the transition amplitude for B and K decays satisfy the $\Delta S = \Delta Q$ rule and CP, CPT invariance. There is no experimental signal of the violation $\Delta S = \Delta Q$ rule. The assumption of CP conservation in the transition amplitude for the B and K decays is valid because $(A(\bar{B}^0 \to J/\psi \bar{K}^0))/(A(B^0 \to J/\psi K^0)) = -1$ to a very high degree and the direct CP violation in the K^0–$\bar{K}^0$ system is very small $(\mathrm{Re}(\epsilon'/\epsilon) \sim 10^{-3})$. We further neglect the small CP violations in K^0–$\bar{K}^0$ mixing; thus $q_K/p_K = 1$.

Under the above assumptions, the decay rate of the cascade decay $B^0 \xrightarrow{t_1} J/\psi + K \xrightarrow{t_2} J/\psi + [\pi^\mp l^\pm \nu]$ is

$$
\begin{aligned}
&\Gamma(B^0, J/\psi + [\pi^\mp l^\pm \nu]) \\
&\equiv \Gamma(B^0 \xrightarrow{t_1} J/\psi + K \xrightarrow{t_2} J/\psi + [\pi^\mp l^\pm \nu]) \\
&\propto \mathrm{e}^{-\Gamma_B t_1} \left\{ \mathrm{e}^{-\Gamma_S t_2} [1 + \sin\phi_0 \sin\Delta m_B t_1 \right. \\
&+\mathrm{Re}\theta' \cos\phi_0 (1 - \cos\Delta m_B t_1) + \mathrm{Im}\theta' \sin\Delta m_B t_1 \\
&+\mathrm{Im}\theta' \sin\phi_0 (1 - \cos\Delta m_B t_1) - \phi'(1 - \cos\Delta m_B t_1) \\
&-\phi' \sin\phi_0 \sin\Delta m_B t_1] + \mathrm{e}^{-\Gamma_L t_2} [1 - \sin\phi_0 \sin\Delta m_B t_1 \\
&-\mathrm{Re}\theta' \cos\phi_0 (1 - \cos\Delta m_B t_1) + \mathrm{Im}\theta' \sin\Delta m_B t_1 \\
&-\mathrm{Im}\theta' \sin\phi_0 (1 - \cos\Delta m_B t_1) - \phi'(1 - \cos\Delta m_B t_1) \\
&+\phi' \sin\phi_0 \sin\Delta m_B t_1] \\
&\pm 2\mathrm{e}^{-\frac{1}{2}(\Gamma_S + \Gamma_L) t_2} [\cos\Delta m_B t_1 \cos\Delta m_K t_2 \\
&+ \cos\phi_0 \sin\Delta m_B t_1 \sin\Delta m_K t_2 \\
&-\mathrm{Re}\theta' \sin\phi_0 (1 - \cos\Delta m_B t_1) \\
&+\mathrm{Im}\theta' \sin\Delta m_B t_1 \cos\Delta m_K t_2 \\
&+\mathrm{Im}\theta' \cos\phi_0 (1 - \cos\Delta m_B t_1) \sin\Delta m_K t_2 \\
&+\phi'(1 - \cos\Delta m_B t_1) \cos\Delta m_K t_2 \\
&\left. -\phi' \cos\phi_0 \sin\Delta m_B t_1 \sin\Delta m_K t_2] \right\}.
\end{aligned} \tag{25}
$$

Similarly, the decay rate of the cascade decay $\bar{B}^0 \xrightarrow{t_1} J/\psi + K \xrightarrow{t_2} J/\psi + [\pi^\pm l^\mp \nu]$ is

$$
\begin{aligned}
&\Gamma(\bar{B}^0, J/\psi + [\pi^\pm l^\mp \nu]) \\
&\equiv \Gamma(\bar{B}^0 \xrightarrow{t_1} J/\psi + K \xrightarrow{t_2} J/\psi + [\pi^\pm l^\mp \nu]) \\
&= \Gamma(B^0 \xrightarrow{t_1} J/\psi + K \xrightarrow{t_2} J/\psi \\
&+[\pi^\mp l^\pm \nu])(\theta' \to -\theta', \phi_0 \to -\phi_0, \phi' \to -\phi').
\end{aligned} \tag{26}
$$

Because we have neglected the small CP violating effects in the K system, only $K_S \to 2\pi$ and $K_L \to 3\pi$ are possible. The decay rate for the cascade decays of $f = 2\pi$ and $f = 3\pi$ are

$$
\begin{aligned}
&\Gamma(\overset{(-)}{B}{}^0, J/\psi + [2\pi]) \equiv \Gamma(\overset{(-)}{B}{}^0 \xrightarrow{t_1} J/\psi + K_S \xrightarrow{t_2} J/\psi + [2\pi]) \\
&\propto 4\mathrm{e}^{-\Gamma_B t_1} \{ \mathrm{e}^{-\Gamma_S t_2} [1 \overset{(-)}{+} \sin\phi_0 \sin\Delta m_B t_1 \\
&\overset{(-)}{+} \mathrm{Re}\theta' \cos\phi_0 (1 - \cos\Delta m_B t_1) \overset{(-)}{+} \mathrm{Im}\theta' \sin\Delta m_B t_1 \\
&+\mathrm{Im}\theta' \sin\phi_0 (1 - \cos\Delta m_B t_1) \overset{(+)}{-} \phi'(1 - \cos\Delta m_B t_1) \\
&-\phi' \sin\phi_0 \sin\Delta m_B t_1]\}
\end{aligned} \tag{27}
$$

and

$$
\begin{aligned}
&\Gamma(\overset{(-)}{B}{}^0, J/\psi + [3\pi]) \equiv \Gamma(\overset{(-)}{B}{}^0 \xrightarrow{t_1} J/\psi + K_L \xrightarrow{t_2} J/\psi + [3\pi]) \\
&\propto 4\mathrm{e}^{-\Gamma_B t_1} \{ \mathrm{e}^{-\Gamma_S t_2} [1 \overset{(+)}{-} \sin\phi_0 \sin\Delta m_B t_1 \\
&\overset{(+)}{-} \mathrm{Re}\theta' \cos\phi_0 (1 - \cos\Delta m_B t_1) \overset{(-)}{+} \mathrm{Im}\theta' \sin\Delta m_B t_1 \\
&+\mathrm{Im}\theta' \sin\phi_0 (1 - \cos\Delta m_B t_1) \overset{(+)}{-} \phi'(1 - \cos\Delta m_B t_1) \\
&+\phi' \sin\phi_0 \sin\Delta m_B t_1]\}.
\end{aligned} \tag{28}
$$

4 The determination of the parameter θ and ϕ

4.1 ϕ'

From (16) and [5],

$$A_T(t) = \frac{\Gamma(B^0(t) \to \bar{X}l^-\nu) - \Gamma(\bar{B}^0(t) \to Xl^+\nu)}{\Gamma(B^0(t) \to \bar{X}l^-\nu) + \Gamma(\bar{B}^0(t) \to Xl^+\nu)} = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} = -2\phi', \quad (29)$$

where N^{++}, N^{--} are dilepton events of the same sign.

ϕ' can be measured by the semileptonic decays of the B mesons or by dilepton ratios of the same sign suggested in [5].

4.2 Imθ'

From (16) and [5],

$$A_{CPT}(t) = \frac{\Gamma(B^0(t) \to Xl^+\nu) - \Gamma(\bar{B}^0(t) \to \bar{X}l^-\nu)}{\Gamma(B^0(t) \to Xl^+\nu) + \Gamma(\bar{B}^0(t) \to \bar{X}l^-\nu)} = \frac{N^{+-} - N^{-+}}{N^{+-} + N^{-+}} = \frac{2\mathrm{Im}\theta' \sin\Delta m_B t}{1 + \cos\Delta m_B t}, \quad (30)$$

where N^{+-}, N^{-+} are dilepton events of opposite signs.

Thus, Imθ' can be measured by the semileptonic decays of the B mesons or by dilepton ratios of opposite signs. For the dileptonic decays in (29) and (30), they correspond to the case of $C = -1$ where C is the charge conjugation number of the $B^0\bar{B}^0$ pair.

Another method is to conclude from (27) and (28)

$$\frac{\Gamma(B^0, J/\psi + [2\pi]) - \Gamma(\bar{B}^0, J/\psi + [2\pi])}{\Gamma(B^0, J/\psi + [2\pi]) + \Gamma(\bar{B}^0, J/\psi + [2\pi])} + \frac{\Gamma(B^0, J/\psi + [3\pi]) - \Gamma(\bar{B}^0, J/\psi + [3\pi])}{\Gamma(B^0, J/\psi + [3\pi]) + \Gamma(\bar{B}^0, J/\psi + [3\pi])} = 2[\mathrm{Im}\theta' \sin\Delta m_B t_1 - \phi'(1 - \cos\Delta m_B t_1)]. \quad (31)$$

Using the known ϕ' value from the semileptonic decays or the dilepton ratios, Imθ' can be determined by (31). But this method is not good experimentally because their branching ratios are smaller than the semileptonic decays.

4.3 $\sin\phi_0$ and the absolute value of the $\cos\phi_0$ and Reθ'

From (27) and (28) we can obtain the time-dependent asymmetry of the decay rates

$$\frac{[\Gamma(B^0, J/\psi + [2\pi]) - \Gamma(\bar{B}^0, J/\psi + [2\pi])]}{[\Gamma(B^0, J/\psi + [2\pi]) + \Gamma(\bar{B}^0, J/\psi + [2\pi])]} - \frac{[\Gamma(B^0, J/\psi + [3\pi]) - \Gamma(\bar{B}^0, J/\psi + [3\pi])]}{[\Gamma(B^0, J/\psi + [3\pi]) + \Gamma(\bar{B}^0, J/\psi + [3\pi])]} = 2[\sin\phi_0 \sin\Delta m_B t_1 + \mathrm{Re}\theta' \cos\phi_0(1 - \cos\Delta m_B t_1)]. \quad (32)$$

The asymmetry of (33) is twice the usual CP asymmetry in the decay of $B \to J/\psi K_S$ because we have used the decay mode of $B \to J/\psi K_L$ to double the asymmetry. We will discuss the problem caused by the detection of K_L later.

There are two contributions in the asymmetry of (33). The $\sin\Delta m_B t_1$ term is an odd function of time while the $(1-\cos\Delta m_B t_1)$ term is an even function. These two terms can be distinguished experimentally by measuring the decay time order of the B^0 and $\bar{B}^0$ decays. The details of this method are given in [16]. Here we only use this method to distinguish the $\sin\Delta m_B t_1$ and $(1 - \cos\Delta m_B t_1)$ terms.

Like [16], define two asymmetries

$$a_-(f,t) = \frac{(\bar{\Gamma} + \tilde{\Gamma}) - (\Gamma + \tilde{\bar{\Gamma}})}{(\bar{\Gamma} + \tilde{\Gamma}) + (\Gamma + \tilde{\bar{\Gamma}})},$$

$$a_+(f,t) = \frac{(\bar{\Gamma} + \tilde{\bar{\Gamma}}) - (\Gamma + \bar{\Gamma})}{(\bar{\Gamma} + \tilde{\bar{\Gamma}}) + (\Gamma + \bar{\Gamma})},$$

where Γ, $\bar{\Gamma}$, $\tilde{\Gamma}$ and $\tilde{\bar{\Gamma}}$ are defined in [16], and the subscript $(-)$ or $(+)$ denotes an odd or even function of time. The measurement of the asymmetry $a_-(f,t)$ requires measuring the decay time order.

Thus the two terms of (33) can be distinguished by

$$A_-(t_1) = a_-(f_1,t_1) - a_-(f_2,t_1) = 2\sin\phi_0 \sin\Delta m_B t_1 \quad (33)$$

and

$$A_+(t_1) = a_+(f_1,t_1) - a_+(f_2,t_1) = 2\mathrm{Re}\theta' \cos\phi_0(1 - \cos\Delta m_B t_1), \quad (34)$$

where f_1 and f_2 represent the final states $J/\psi + [2\pi]$ and $J/\psi + [3\pi]$.

In (34), the asymmetry is often used to measure the direct CP violation when CPT invariance holds. The direct CP violation in $B \to J/\psi K$ decays is of the order of $\mathcal{O}(10^{-3})$. So the measurement of the CPT violation in (34) can only reach an accuracy up to 10^{-2} because of the dilution of direct CP violation in $B \to J/\psi K$ decay and the CP violation in K^0–$\bar{K}^0$ mixing.

From the (33) and (34), the values of $\sin\phi_0$ and Re$\theta' \cos\phi_0$ can be obtained. So $\cos\phi_0$ and Reθ' can be determined except for the ambiguity of their sign. This ambiguity can be solved by the cascade decays where $f = \pi l\nu$.

4.4 The sign of $\cos\phi_0$ and Reθ'

From (25) and (27),

[see (35) and (36) on top of the next page]

where

$$A = 2\mathrm{e}^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_2}[\cos\Delta m_B t_1 \cos\Delta m_K t_2 + \cos\phi_0 \sin\Delta m_K t_2(\sin\Delta m_B t_1$$

$$\frac{\Gamma(B^0,J/\psi+[\pi^-l^+\nu])-\Gamma(B^0,J/\psi+[\pi^+l^-\nu])+\Gamma(\bar{B}^0,J/\psi+[\pi^-l^+\nu])-\Gamma(\bar{B}^0,J/\psi+[\pi^+l^-\nu])}{\Gamma(B^0,J/\psi+[\pi^-l^+\nu])+\Gamma(B^0,J/\psi+[\pi^+l^-\nu])+\Gamma(\bar{B}^0,J/\psi+[\pi^-l^+\nu])+\Gamma(\bar{B}^0,J/\psi+[\pi^+l^-\nu])}$$
$$= \frac{-2\mathrm{e}^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_2}\mathrm{Re}\theta'\sin\phi_0(1-\cos\Delta m_B t_1)}{\{\mathrm{e}^{-\Gamma_S t_2}[1+\mathrm{Im}\theta'\sin\phi_0(1-\cos\Delta m_B t_1)-\phi'\sin\phi_0\sin\Delta m_B t_1] + \mathrm{e}^{-\Gamma_L t_2}[1-\mathrm{Im}\theta'\sin\phi_0(1-\cos\Delta m_B t_1)+\phi'\sin\phi_0\sin\Delta m_B t_1]\}}$$
$$\sim \frac{-2\mathrm{e}^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_2}\mathrm{Re}\theta'\sin\phi_0(1-\cos\Delta m_B t_1)}{\mathrm{e}^{-\Gamma_S t_2}+\mathrm{e}^{-\Gamma_L t_2}} \quad (35)$$

$$\frac{\Gamma(B^0,J/\psi+[\pi^-l^+\nu])-\Gamma(B^0,J/\psi+[\pi^+l^-\nu])-\Gamma(\bar{B}^0,J/\psi+[\pi^-l^+\nu])+\Gamma(\bar{B}^0,J/\psi+[\pi^+l^-\nu])}{\Gamma(B^0,J/\psi+[\pi^-l^+\nu])+\Gamma(B^0,J/\psi+[\pi^+l^-\nu])+\Gamma(\bar{B}^0,J/\psi+[\pi^-l^+\nu])+\Gamma(\bar{B}^0,J/\psi+[\pi^+l^-\nu])}$$
$$= \frac{A}{B} \sim \frac{2\mathrm{e}^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_2}[\cos\Delta m_B t_1\cos\Delta m_K t_2+\cos\phi_0\sin\Delta m_K t_2\sin\Delta m_B t_1]}{\mathrm{e}^{-\Gamma_S t_2}+\mathrm{e}^{-\Gamma_L t_2}}, \quad (36)$$

$$- \mathrm{Im}\theta'(1-\cos\Delta m_B t_1)+\phi'\sin\Delta m_B t_1)$$
$$- \mathrm{Im}\theta'\sin\Delta m_B t_1\cos\Delta m_K t_2-\phi'(1-\cos\Delta m_B t_1)],$$
$$B = \mathrm{e}^{-\Gamma_S t_2}[1+\mathrm{Im}\theta'\sin\phi_0(1-\cos\Delta m_B t_1) - \phi'\sin\phi_0\sin\Delta m_B t_1] + \mathrm{e}^{-\Gamma_L t_2}[1-\mathrm{Im}\theta'\sin\phi_0(1-\cos\Delta m_B t_1) + \phi'\sin\phi_0\sin\Delta m_B t_1].$$

From (35) and (36), the sign of $\cos\phi_0$ and $\mathrm{Re}\theta'$ can be measured. Actually, we do not need to use (35) since we know the value of $\mathrm{Re}\theta'\cos\phi_0$.

5 Feasibility and discussions

In order to meet the goal of a three standard deviation measurement for the B^0–$\bar{B}^0$ mixing parameter θ and ϕ, the relation between the number of B^0–$\bar{B}^0$ pairs and the asymmetry is [17]

$$N_{B^0\bar{B}^0} = \frac{1}{B\epsilon_r\epsilon_t[(1-2W)d\cdot\delta A]^2}, \quad (37)$$

where $\delta A = A/3$, A is the asymmetry of the decay ratios, B is the branching ratio of the decay, ϵ_r is the reconstruction efficiency of the final state f, ϵ_t is the tagging efficiency, W is the fraction of incorrect tags, and d is the dilution factor which takes into account the loss in asymmetry due to fitting, time integration, and/or the mixing of the tagged decay.

Sometimes, another relation is used:

$$N_{\mathrm{eff}} = \frac{1}{[(1-2W)d\cdot\delta A]^2}, \quad (38)$$

where N_{eff} is the effective number of decay events.

The B-factories can accumulate 1.8×10^8 B–$\bar{B}$ pairs every year [19], and the effective event number of (38) for $B\to J/\psi K_S$ is taken to be 2.7×10^5 in LHC-B [20]. Table 1 and Table 2 give some parameters [17] and the minimum asymmetries (lower bound) which can be achieved in a B-factory and at LHC-B.

Now we discuss how to determine the two complex phases θ and ϕ.

(1) ϕ' and $\mathrm{Im}\theta'$ can be measured in the semileptonic decays and dileptonic decays given in (29) and (30). Semileptonic decays have a larger branching ratio but a smaller detection efficiency, while dileptonic decays have a larger detection efficiency but a smaller branching ratio than semileptonic decays. In a B-factory (with 1.8×10^8 $B^0\bar{B}^0$ pairs per year), the ϕ' and $\mathrm{Im}\theta'$ can be measured to an accuracy of 2×10^{-3} for semileptonic decays and 7×10^{-3} for dileptonic decays at the 3σ level. In LHC-B (with 4×10^{12} $b\bar{b}$ pairs every snow mass year), the ϕ' and $\mathrm{Im}\theta'$ can be measured to an accuracy of 10^{-5} for semileptonic decays and 4×10^{-5} for dileptonic decays with a 3σ standard deviation.

(2) $\mathrm{Re}\theta'\cos\phi_0$ can clearly be determined from (34). In (33) the decay of $K_L\to3\pi$ is used in order to increase the asymmetry factor and reduce the ambiguity or error caused by the unknown ϕ' and $\mathrm{Im}\theta'$. The K_L detection is a challenge for the experiment. In [18], one idea to catch K_L, namely by using Fe sampling after the electromagnetic calorimeter, is suggested. In a B-factory, the accuracy of measuring $\mathrm{Re}\theta'\cos\phi_0$ can reach 0.07. So it is likely that $\mathrm{Re}\theta'\cos\phi_0$ cannot be measured in a B-factory. In LHC-B, the accuracy of measuring $\mathrm{Re}\theta'\cos\phi_0$ can reach 0.01. We have no confidence that $\mathrm{Re}\theta'\cos\phi_0$ can be measured clearly with a 3σ standard deviation in LHC-B. If it can be measured, $\mathrm{Re}\theta'$ must be larger than 0.01. This will be the largest of the *CPT* violation effects.

(3) $\sin\phi_0$ is usually suggested to be measured in $B\to J/\psi K_S(\to\pi^+\pi^-)$ which gives $A(t)=\sin\phi_0\sin\Delta m_B t$ in the standard model. If the *CPT* violating effects are considered, the asymmetry is modified to (18). In order to cancel the errors caused by ϕ' and $\mathrm{Im}\theta'$, we use (33) to determine $\sin\phi_0$.

(4) $\sin\phi_0$ can be measured in a B-factory and at LHC-B as discussed above, but the value of ϕ_0 has two ambiguities. If $\mathrm{Re}\theta'\cos\phi_0$ can be measured, the only ambiguity is the sign of $\cos\phi_0$. This can be solved by measuring the ratios of the cascade decays $B\to J/\psi+K\to J/\psi+[\pi l\nu]$ given in (36). Because the very small branching ratio of

Table 1. Branching ratios and reconstruction efficiencies for the cascade decays and semileptonic decays

Decay mode	Branching ratio B	Reconstruction effeciency ϵ_r
$B \to J/\psi K_S \to J/\psi + [2\pi]$	5×10^{-4}	0.61
$B \to J/\psi K_L \to J/\psi + [3\pi]$	$5 \times 10^{-4} \times 1/3$	0.4
$B \to J/\psi K_S \to J/\psi + [\pi l\nu]$	$5 \times 10^{-4} \cdot 1.2 \times 10^{-3}$	0.61
$B \to J/\psi K_L \to J/\psi + [\pi l\nu]$	$5 \times 10^{-4} \times 2/3$	0.4
$B \to l\nu + X$	0.1	1
$J/\psi \to l^+l^-$	0.14	-

Tag efficiencies and asymmetry dilution at a B-factory and at LHC-B

	at B-factory	At LHC-B
Tag efficiency ϵ_t	0.48	0.61
Asymmetry dilution d	0.61 (for B_d)	0.61

Table 2. Comparison between the minimum of asymmetries with 3σ standard deviation at B-factory and LHC-B

Decay mode	Asymmetry A	
	at B-factory	at LHC-B
$B \to J/\psi K_S \to J/\psi + [2\pi]$	0.08	1.2×10^{-2}
$B \to J/\psi K_L \to J/\psi + [3\pi]$	0.17	2.6×10^{-2}
$B \to J/\psi K_S \to J/\psi + [\pi l\nu]$	1.44	0.27
$B \to J/\psi K_L \to J/\psi + [\pi l\nu]$	0.06	0.01
$B \to l\nu + X$	1.7×10^{-3}	1×10^{-5}
$B\bar{B} \to l^+l^-$	7×10^{-3}	4×10^{-5}

the decay $B \to J/\psi + K \to J/\psi + [\pi l\nu]$ in $t_2 \leq 2\tau_S$, the determination of the sign of $\cos\phi_0$ can only be done in LHC-B. Table 2 tells that the lower bound of measuring the asymmetry in cascade decay is 0.2. This is possible because $|\cos\phi_0|\rangle 0.5$; we have taken $0.3 \leq \sin\phi_0 \leq 0.88$ [21] for our estimation.

In conclusion, the cascade decays provide an elegant and beautiful place to study the *CPT* violation caused by the B^0–$\bar{B}^0$ mixing.

Acknowledgements. This work is supported in part by National Natural Science Foundation of China and the Grant of State Commission of Science and Technology of China.

References

1. CPLEAR Collaboration (A. Angelopoulos et al.) Phys. Lett. B **444**, 43 (1998)
2. Fermilab KTeV collaboration, seminar talk by P. Shawhan
3. CPLEAR Collaboration (A. Angelopoulos et al.) Phys. Lett. B **444**, 52 (1998)
4. V.A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998)
5. M. Kobayashi, A.I. Sanda, Phys. Rev. Lett. **69**, 3139 (1992)
6. Z.Z. Xing, Phys. Rev. D 50 (1994) R2957; Phys. Lett. B **450**, 202 (1999)
7. Y.I. Azimov, Phys. Rev. D **42**, 3705 (1990)
8. B. Kayser, hep-ph/9709382
9. C.C. Meca, J.P. Silva, Phys. Rev. Lett. **81**, 1377 (1998)
10. PDG98, Eur. Phys. J. C 3 (1998)
11. Y. Nir, H.R. Quinn, Annu. Rev. Nucl. Part. Sci. **42**, 211 (1992)
12. T.D. Lee, C.S. Wu, Annu. Rev. Nucl. Sci **16**, 471 (1966)
13. Current physics sources and comments, Vol. 5, *CP* Violation, edited by L. Wolfenstein
14. M. Hayakawa, A.I. Sanda, DPNU 93-05
15. B. Kayser, L. Stodolsky, Phys. Lett. B **359**, 343 (1995)
16. M. Gronau, Phys. Rev. Lett. **63**, 1451 (1989)
17. SLAC-353, LBL-27856, CALT-1588, October, 1992
18. SLAC-400, 529, November, 1992, B factories
19. The BaBar Collaboration, Technical Design Report SLAC-R-95-457, March, 1995
20. T. Nakada, hep-ex/9502005
21. Y. Nir, hep-ph/9709301, or in the Proceedings of the 18th International Symposium on Lepton Photon Interactions, Hamberg, Germany, July 28–August 1 1997